EXERCISE – III SUBJECTIVE QUESTIONS

1. The third term of an A.P. is 18, and the seventh term is 30 ; find the sum of 17 terms.

2. Find the number of integers between 100 & 1000 that are
(i) divisible by 7

(ii) not divisible by 7

3. Find the sum of all those integers between 100 and 800 each of which on division by 16 leaves the remainder 7.

4. The sum of three numbers in A.P. is 27, and their product is 504, find them.

5. If a, b, c are in A.P., then show that
(i) $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are also in A.P.

(ii) $b + c - a$, $c + a - b$, $a + b - c$ are in A.P.

6. The continued product of three numbers in G.P. is 216, and the sum of the products of them in pairs is 156, find the numbers.

7. If the p^{th}, q^{th}, r^{th} terms of a G.P. be a, b, c respectively, prove that $a^{q-r}\, b^{r-p}\, c^{p-q} = 1$.

8. The sum of three numbers which are consecutive terms of an A.P. is 21. If the second number is reduced by 1 & the third is increased by 1, we obtain three consecutive terms of a G.P., find the numbers.

9. The sum of infinite number of terms of G.P. is 4 and the sum of their cubes is 192. Find the series.

10. Sum the following series

(i) $1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots$ to n terms.

(ii) $1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \ldots\ldots$ to infinity.

11. Find the sum of n terms of the series the r^{th} term of which is $(2r + 1)2^r$.

12. Find the 4^{th} term of an H.P. whose 7^{th} term is $\frac{1}{20}$ and 13^{th} term is $\frac{1}{38}$.

13. The arithmetic mean of two numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$. Find the two numbers.

14. Using the relation A.M. ≥ G.M. prove that

(i) $\tan\theta + \cot\theta \geq 2$; if $0 < \theta < \frac{\pi}{2}$

(ii) $(x^2y + y^2z + z^2x)(xy^2 + yz^2 + zx^2) > 9x^2y^2z^2$. Where x,y,z are different real no.

(iii) $(a + b) \,.\, (b + c) \,.\, (c + a) \geq 8abc$; if a, b, c are positive real numbers.

15. Find the sum of the n terms of the series whose nth term is
(i) $n(n + 2)$

(ii) $3^n - 2^n$

16. The sum of the first ten terms of an AP is 155 & the sum of first two terms of a GP is 9. The first term of the AP is equal to the common ratio of the GP & the first term of the GP is equal to the common difference of the AP. Find the two progressions.

17. Find the sum in the n^{th} group of sequence,
(i) 1, (2, 3); (4, 5, 6, 7); (8, 9,........, 15) ; ....

(ii) (1), (2, 3, 4), (5, 6, 7, 8, 9), ...

18. Find the sum of the series

$\frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + \frac{5555}{(13)^4}$ .... up to ∞

19. If $0 < x < \pi$ and the expression
$\exp\{(1 + |\cos x| + \cos^2 x + |\cos^3 x| + \cos^4 x + \ldots\ldots \text{ upto } \infty) \log_e 4\}$
satisfies the quadratic equation $y^2 - 20y + 64 = 0$ the find the value of x.

20. In a circle of radius R a square is inscribed, then a circle is inscribed in the square, a new sauare in the circle and so on for n times. Find the limit of the sum of areas of all the circle and the limit of the sum of areas of all the squares as $n \to \infty$.

21. If a, b, c are sides of triangle then prove that
(i) $b^2c^2 + c^2a^2 + a^2b^2 \geq abc\,(a + b + c)$

(ii) $(a + b + c)^3 > 27\,(a + b - c)\,(c + a - b)\,(b + c - a)$

22. Sum the following series to n terms and to infinity

(i) $\sum_{r=1}^{n} r(r + 1)\,(r + 2)\,(r + 3)$

(ii) $\frac{n}{1.2.3} + \frac{n-1}{2.3.4} + \ldots\ldots + \frac{1}{n(n+1)(n+2)}$.

23. Sum of the series to n terms and to infinity :

$1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + \ldots\ldots\infty.$

24. In an A.P. of which 'a' is the Ist term, if the sum of the Ist 'p' terms is equal to zero, show that the sum of the next 'q' terms is $-\frac{a(p+q)q}{p-1}$.

25. The number of terms in an A.P. is even; the sum of the odd terms is 24, sum of the even terms 30, and the last term exceeds the first by 10½; find the number of terms.

26. A man arranges to pay off debit of Rs. 3600 by 40 annual instalments which form an arithmetic series. When 30 of the instalments are paid he dies leaving a third of the debt unpaid. Find the value of the first instalment.

27. If the p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, show that
$(q - r)a + (r - p)\,b + (p - q)\,c = 0.$

28. If b is the harmonic mean between a and c prove that $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$.

29. Circles are inscribed in the acute angle α so that every neighbouring circles touch each other. If the radius of the first circle is R then find the sum of the radii of the first n circles in terms of R and α.

30. The first term of arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression conicide with the first and the ninth term of the arithmetic progression. Find the seventh term of the geometric progression.